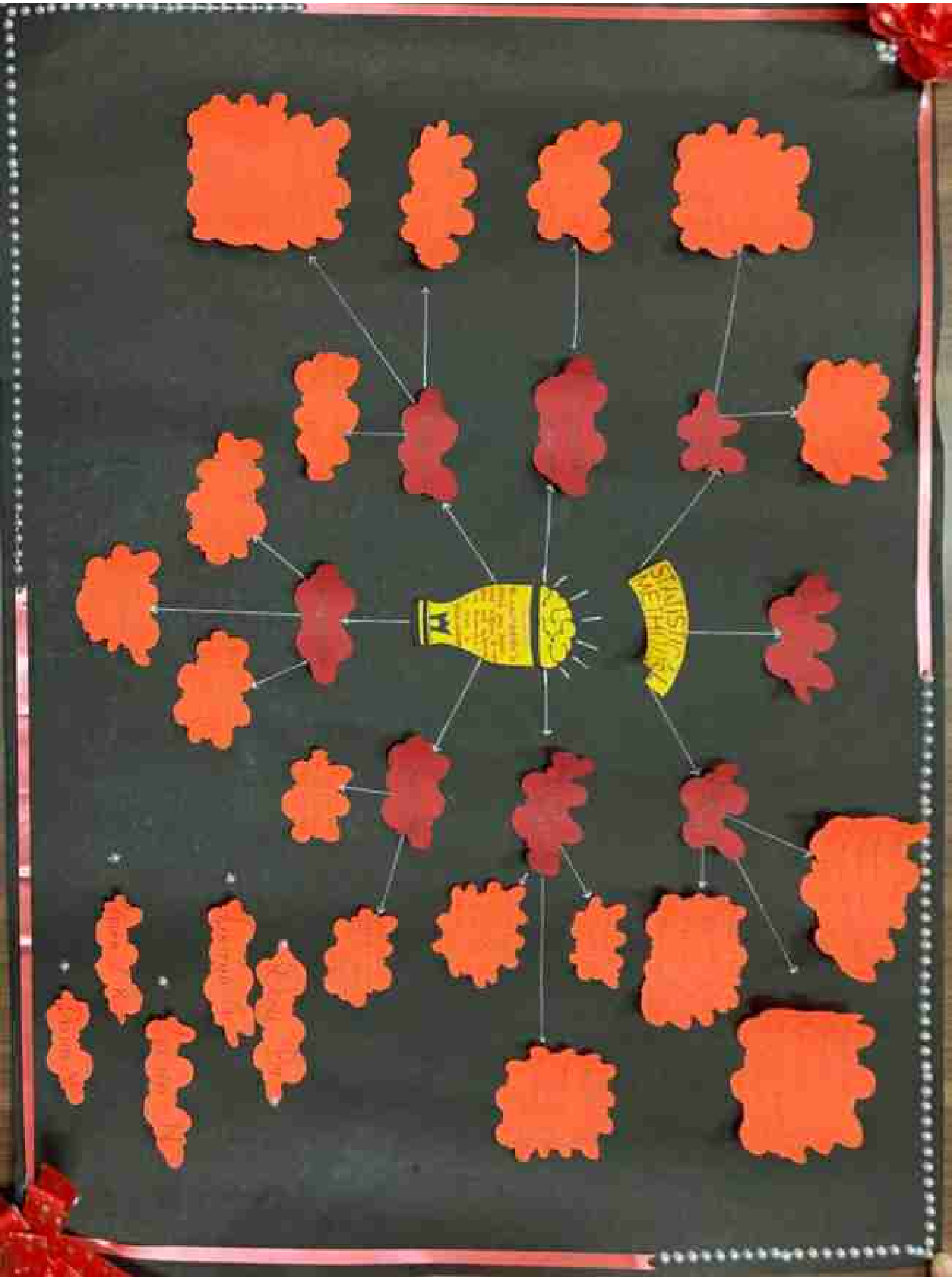
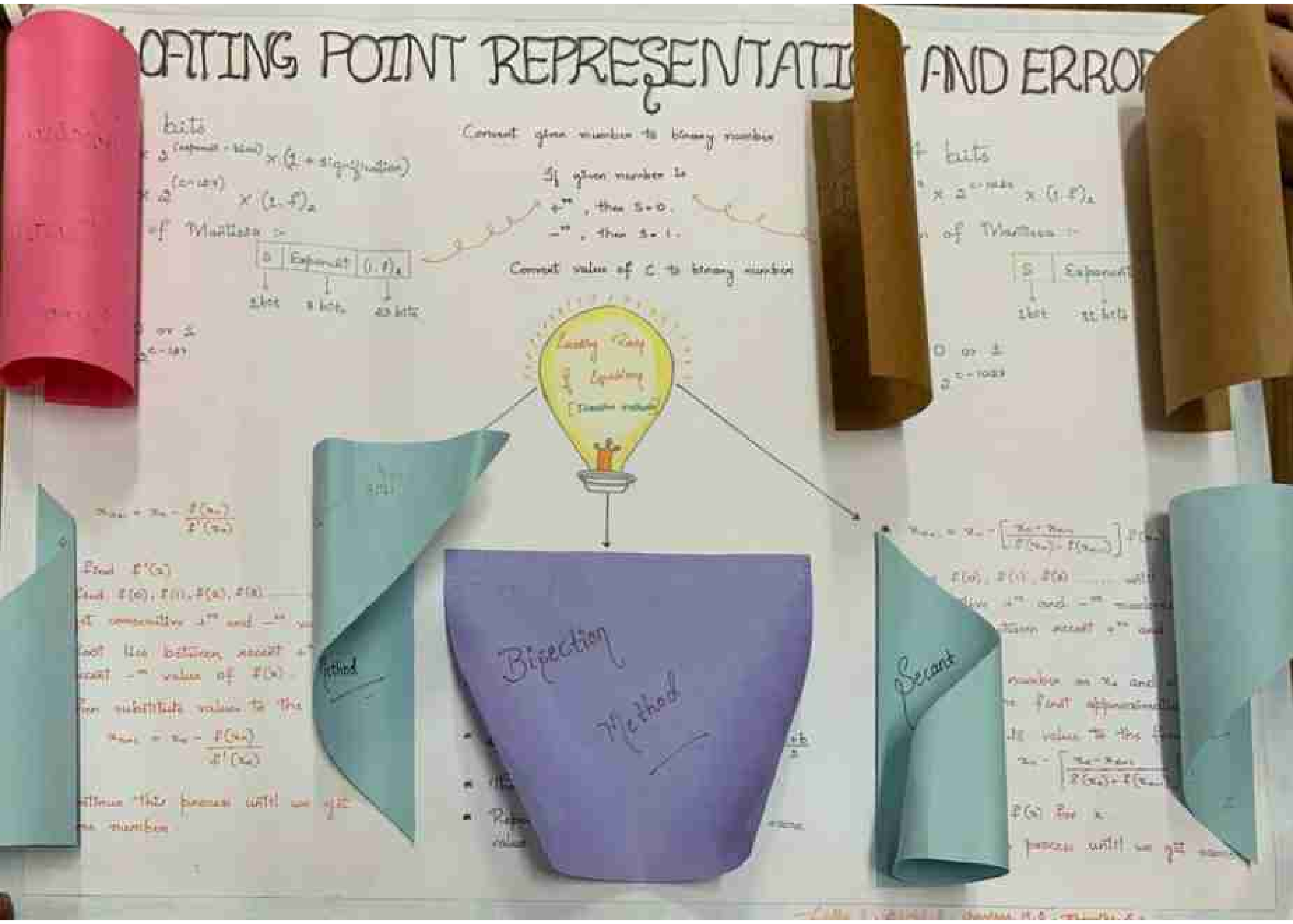
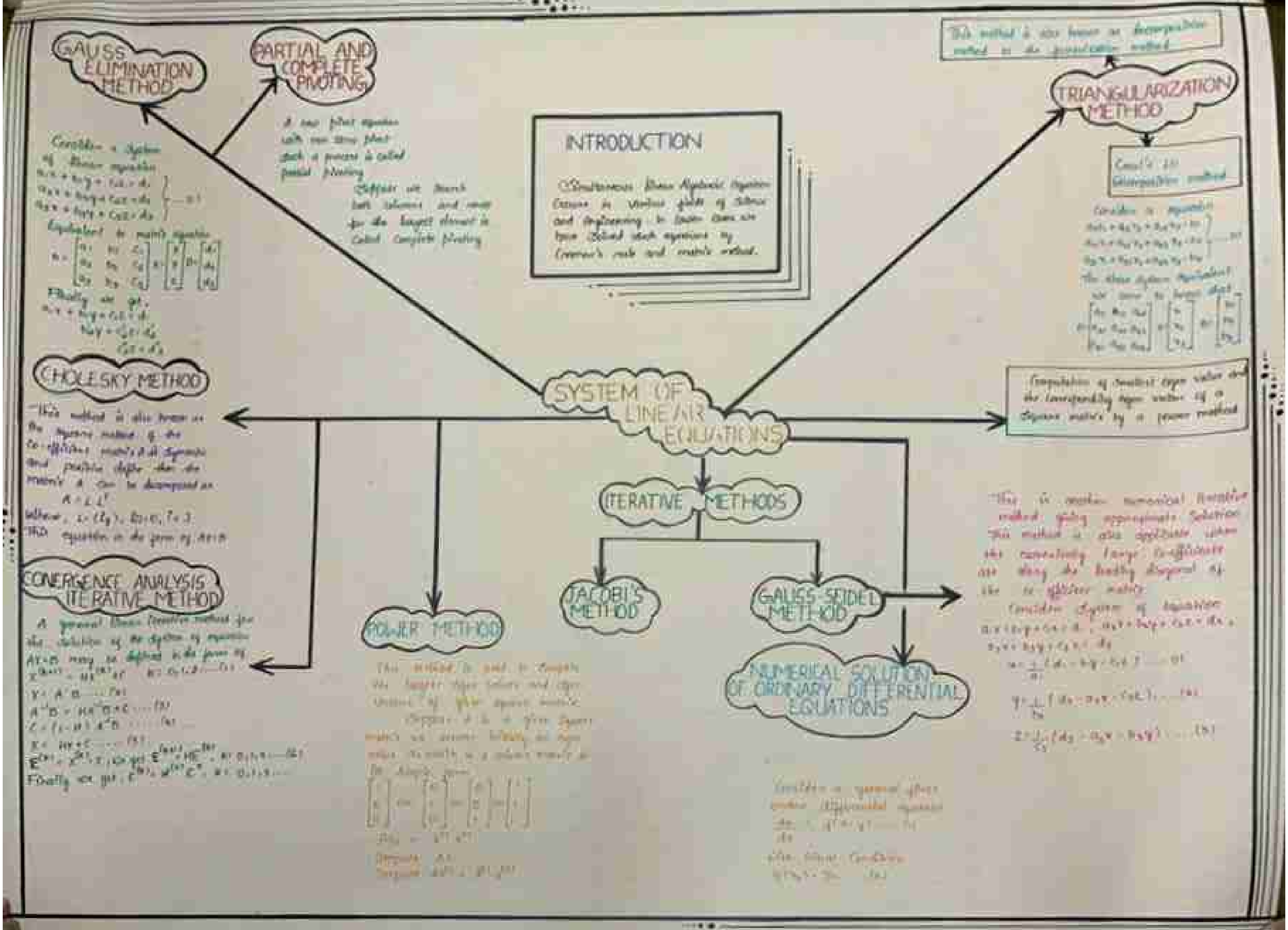



OATING POINT REPRESENTATI
AND ERRO
Convert given number to binary number
If given number is
+ve, then S = 0.
−ve, then S = 1.
Convert value of E to binary number
Bisection
Method
Method
Secant


SYSTEM OF LINEAR EQUATIONS
INTRODUCTION
Simultaneous linear algebraic equations occur in various fields of science and engineering. In lower cases we have solved such equations by Cramer's rule and matrix method.
GAUSS ELIMINATION METHOD
Consider a system of linear equations
Equivalent to matrix equation
Finally we get,
PARTIAL AND COMPLETE PIVOTING
A new pivot equation with non zero pivot such a process is called partial pivoting
Suppose we search both columns and rows for the largest element is called complete pivoting
TRIANGULARIZATION METHOD
This method is also known as decomposition method or the factorization method.
Crout's LU decomposition method
Consider the equations
CHOLESKY METHOD
This method is also known as the square root method. If the coefficient matrix A is symmetric and positive definite then the matrix A can be decomposed as
$A = LL^T$
Computation of smallest eigen value and the corresponding eigen vectors of a square matrix by a power method
ITERATIVE METHODS
JACOBI'S METHOD
GAUSS-SEIDEL METHOD
This is another numerical iterative method giving approximate solution. This method is also applicable when the comparatively large coefficients are along the leading diagonal of the co-efficient matrix.
Consider system of equations
CONVERGENCE ANALYSIS ITERATIVE METHOD
A general linear iterative method for the solution of the system of equation $AX = B$ may be defined in the form of
Finally we get,
POWER METHOD
This method is used to compute the largest eigen values and eigen vectors of given square matrix.
NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS
Consider a general first order differential equation
with initial condition

System of Linear Equations

Gauss Elimination Method [Direct Method]

Consider the system of linear equations

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

Write the equation as $AX = B$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Let us consider the augmented matrix $(A|B)$ of the given system as follows

$$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

Step 1: (row) to make the elements ... zero by elementary row transformation, this transforms $(A|B)$ into the form

$$(A|B) = \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ 0 & b_2 & c_2 & d_2 \\ 0 & b_3 & c_3 & d_3 \end{array}\right]$$

Step 2: (row) to make the element ... zero by the transformation, this transforms into the form

$$(A|B) = \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ 0 & b_2 & c_2 & d_2 \\ 0 & 0 & c_3 & d_3 \end{array}\right]$$

From (2) the system of linear equations is equivalent to

$a_1x + b_1y + c_1z = d_1$
$b_2y + c_2z = d_2$
$c_3z = d_3$

Using the back substitution method, we find the values of x, y, z.

Partial and Complete Pivoting

* The Gauss elimination method can be applied with minor modification. The modification method is called the method of partial pivoting.
* Partial pivoting is the interchanging of rows ... is called full pivoting. It is the interchanging of both rows and columns in order to place a particularly "good" element in the diagonal position prior to a particular operation.
* The resulting modified algorithm is called Gaussian elimination with partial pivoting.

LU Decomposition Method

In this method, the coefficient matrix of the system of equations ...

* Consider the system of equations
$a_{11}x + a_{12}y + a_{13}z = b_1$
$a_{21}x + a_{22}y + a_{23}z = b_2$
$a_{31}x + a_{32}y + a_{33}z = b_3$
* The system (1) is equivalent to matrix form $AX = B$, where ...
* If A is a square matrix and it can be factored as $A = LU$, where L is a lower triangular matrix and U is an upper triangular matrix, then we say that A has an LU decomposition.

where
$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

The (1) becomes $LUX = B$. Let $UX = Y$ ... $LY = B$

* Multiplying and equating, we find the values y_1, y_2, y_3 (forward ...)
* Multiplying and equating and by back substitution, find the values of x, y and z.

Jacobi Iteration Method

Consider system of linear equations

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

Solving for x, y, z respectively, the system can be written in the form

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$$

Let us start the initial approximation x_0, y_0, z_0 for the values of x, y, z. Substituting these in eq. (1) we get first iteration

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$
$$y_1 = \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0)$$
$$z_1 = \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0)$$

Second, iteration values are given by

$$x_2 = \frac{1}{a_1}(d_1 - b_1y_1 - c_1z_1)$$
$$y_2 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_1)$$
$$z_2 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

* This process is repeated till the difference b/w two consecutive approximation is negligible.
* In the initial approximation x_0, y_0, z_0 for the values of unknowns, the initial values chosen as $x_0 = 0, y_0 = 0, z_0 = 0$ if initial approximation is not given.

Gauss-Seidal Iteration Method

* This is a modification of the Jacobi's iteration method.

Consider a system of equations

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

Rewrite the above system as

$$x = \frac{1}{a_1}[d_1 - b_1y - c_1z]$$
$$y = \frac{1}{b_2}[d_2 - a_2x - c_2z]$$
$$z = \frac{1}{c_3}[d_3 - a_3x - b_3y]$$

Let the initial approximation of the solution be x_0, y_0, z_0

First approximation:

$$x_1 = \frac{1}{a_1}[d_1 - b_1y_0 - c_1z_0]$$
$$y_1 = \frac{1}{b_2}[d_2 - a_2x_1 - c_2z_0]$$
$$z_1 = \frac{1}{c_3}[d_3 - a_3x_1 - b_3y_1]$$

Second approximation:

$$x_2 = \frac{1}{a_1}[d_1 - b_1y_1 - c_1z_1]$$
$$y_2 = \frac{1}{b_2}[d_2 - a_2x_2 - c_2z_1]$$
$$z_2 = \frac{1}{c_3}[d_3 - a_3x_2 - b_3y_2]$$

Continue the process till any two consecutive approximations are almost identical.

Power Method

* The power method is used to find the dominant eigenvalue of a matrix and a corresponding eigenvector. To apply the power method to a square matrix A, begin with an initial guess for an eigenvector of the dominant eigenvalue.

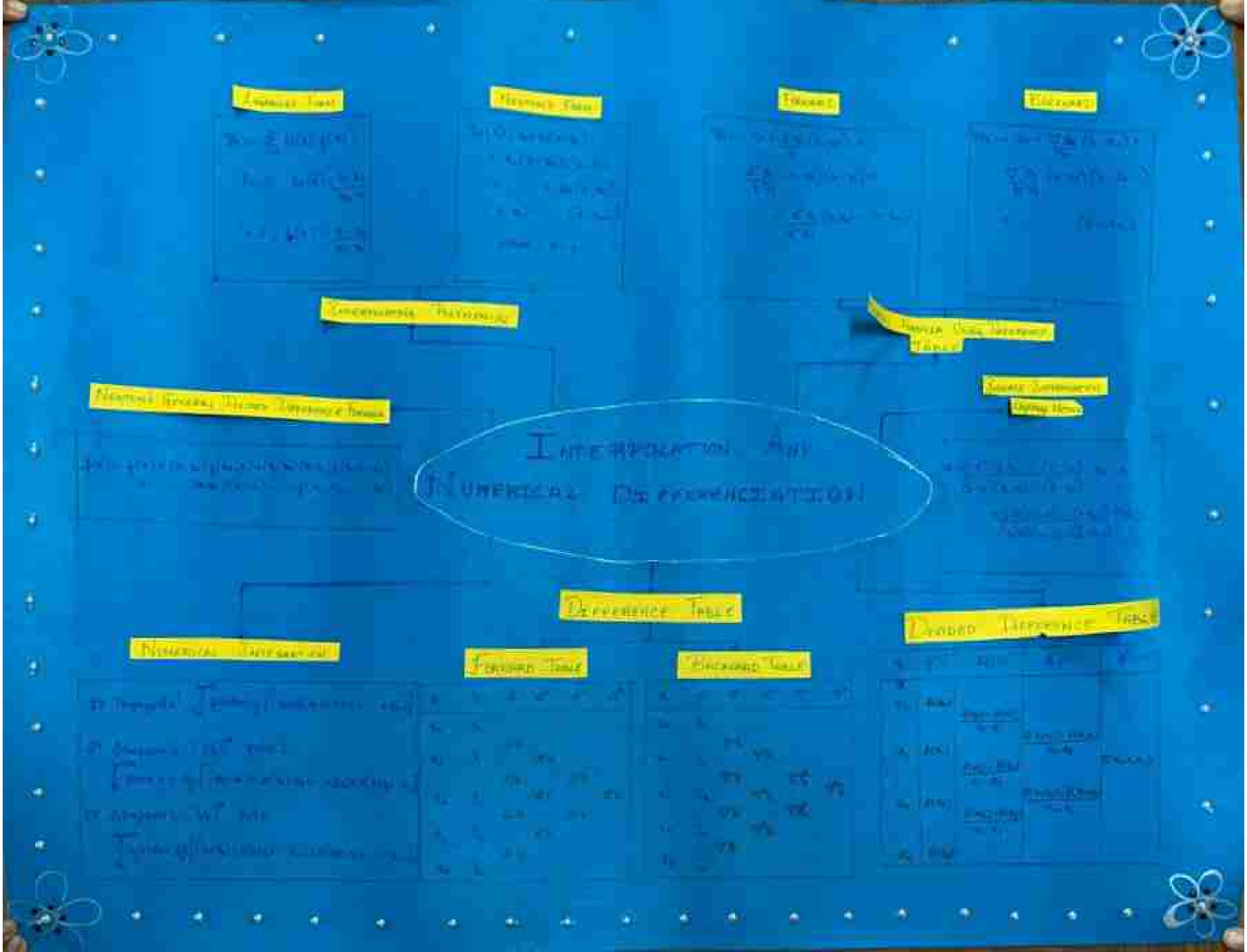
Consider a square matrix A. Let $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

First approximation is $AX_0 = \lambda^{(1)} X^{(1)}$

Second approximation is $AX^{(1)} = \lambda^{(2)} X^{(2)}$

The process for any two consecutive approximations are almost identical.

* Largest eigenvalue (λ^{*}) & corresponding eigenvector (X^{*}) of the given square matrix.


Interpolation And
Numerical Differentiation
Difference Table
Forward Table
Backward Table
Divided Difference Table

FLOATING POINT REPRESENTATION AND ERRORS
DECIMAL
BINARY
OCTAL
HEXADECIMAL
BINARY REPRESENTATION
OCTAL REPRESENTATION
0, 1, 2, 3, 4, 5, 6, 7, 8
HEXA DECIMAL REPRESENTATION
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
DECIMAL REPRESENTATION
The notation i.e, is used for decimal representation.
ERROR
Numerical calculations always involve approximation due to several reasons.
1. MACHINE REPRESENTATION AND ARITHMETIC ERRORS
These errors are inevitable when using floating point arithmetic when using computers or calculators.
2. MODELLING ERRORS:-
MATHEMATICAL MODELLING is a process when mathematical equations are used to represent a physical system.
3. Mathematical error
TYPES OF ERRORS
1. ABSOLUTE AND RELATIVE ERRORS
2. INHERENT ERROR
3. ROUND OFF ERROR
4. TRUNCATION ERROR
1. Absolute error
Approximate value
Relative error:-
2. ROUND OFF ERRORS:-
3. INHERENT ERRORS:-
4. Truncation error:-
Truncation errors are defined as those errors that result from using an approximation in place of an exact mathematical procedure.
FLOATING POINT REPRESENTATION
STANDARD WAY
NORMALIZED WAY
NORMALIZED FLOATING POINT REPRESENTATION
2. FLOATING POINT REPRESENTATION BINARY
3. MACHINE NUMBERS
4. MACHINE EPSILON(E)
NORMALIZED FLOATING POINT REPR
2. DOUBLE PRECISION FLOATING POINT
AVOIDING LOSS OF SIGNIFICANCE
2. Using Taylor series expansion
3. Using Trigonometric identities
4. Range reduction
5. Exponential properties
Algebraic and Transcendental Equations
1. Bisection method
2. Secant method
3. Newton - Raphson method
Tejaswini R
Rithya Shree

NSM
UNIT-3
SYSTEM of LINEAR EQUATION

Dejination :- So far we have studied the methods of solving linear equation Method, "Crammer's rule" and "Matrix Methods". These methods are used when the linear equation are very less in number. When the number of unknown in the system is large, the methods become tedious. To solve the such types of equation we use various other numerical methods which are suited for computer operations.

These numerical methods have 2 types

- Direct Method
  - Gass-Elimination Method
  - Triangularization Method
  - Doolittle Method.
  - Cholesky Method.
- Indirect Method
  - Jacobi's Method.
  - Gauss-Seidel Method.

Gresika. N.
Kumuda. M.
Ramya. M. G.
Ramya. N.

FLOATING POINT REPRESENTATION

Introduction :- The aim of Numerical Method is to provide efficient methods for obtaining numerical answers to such problems with the above advent of the modern high speed electronic digital computer, our main concern is to provide computer oriented, efficient and reliable numerical methods for solving problems arising in different areas of higher mathematics.

Normal floating point

In the decimal system any real number x (other than zero) can be represented in Normalised floating point form as

$x = \pm 0.d_1d_2d_3 \ldots \times 10^{n}$

Ex :- 58.2345 → 0.582345×10^2

0.003567 → 0.3567×10^{-2}

Range of integer → −32768 to 32767

Range of floating point → $1.2e^{-38}$ to $3.4e^{38}$

Subtract the floating point number

1)
0.36143447×10^7
$-\ 0.36132346 \times 10^7$
0.00011101×10^7

2)
0.2350×10^3
$-\ 0.0356 \times 10^3$
0.1994×10^3

Single Precision

$(-1)^s \times 2^{(\text{exponent} - \text{bias})} \times (1 + \text{significant})$

For single precision $e = \pm 127$

Sign bit : 0 - positive
1 - Negative

Represent the Number

s	e	f
1	8	23

= 32 bits

Double Precision

$(-1)^s \times 2^{(\text{exponent} - \text{bias})} \times (1 \times \text{Significant})$

For Double Precision $e = \pm 1023$

Sign bit : 0 - positive
1 - Negative

Represent the Number

s	e	f
1	11	52

= 64 bits

Secant Method

The secant method converges for the approximated root similar to Newton-Raphson Method

$$x_{n+1} = x_n - \left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right] f(x_n)$$

Newton's Method :

The procedure known as Newton's method is also called the Newton-Raphson Iteration

$x_{n+1} = x_n - h$

where

$$h = \frac{f(x_n)}{f'(x_n)}$$

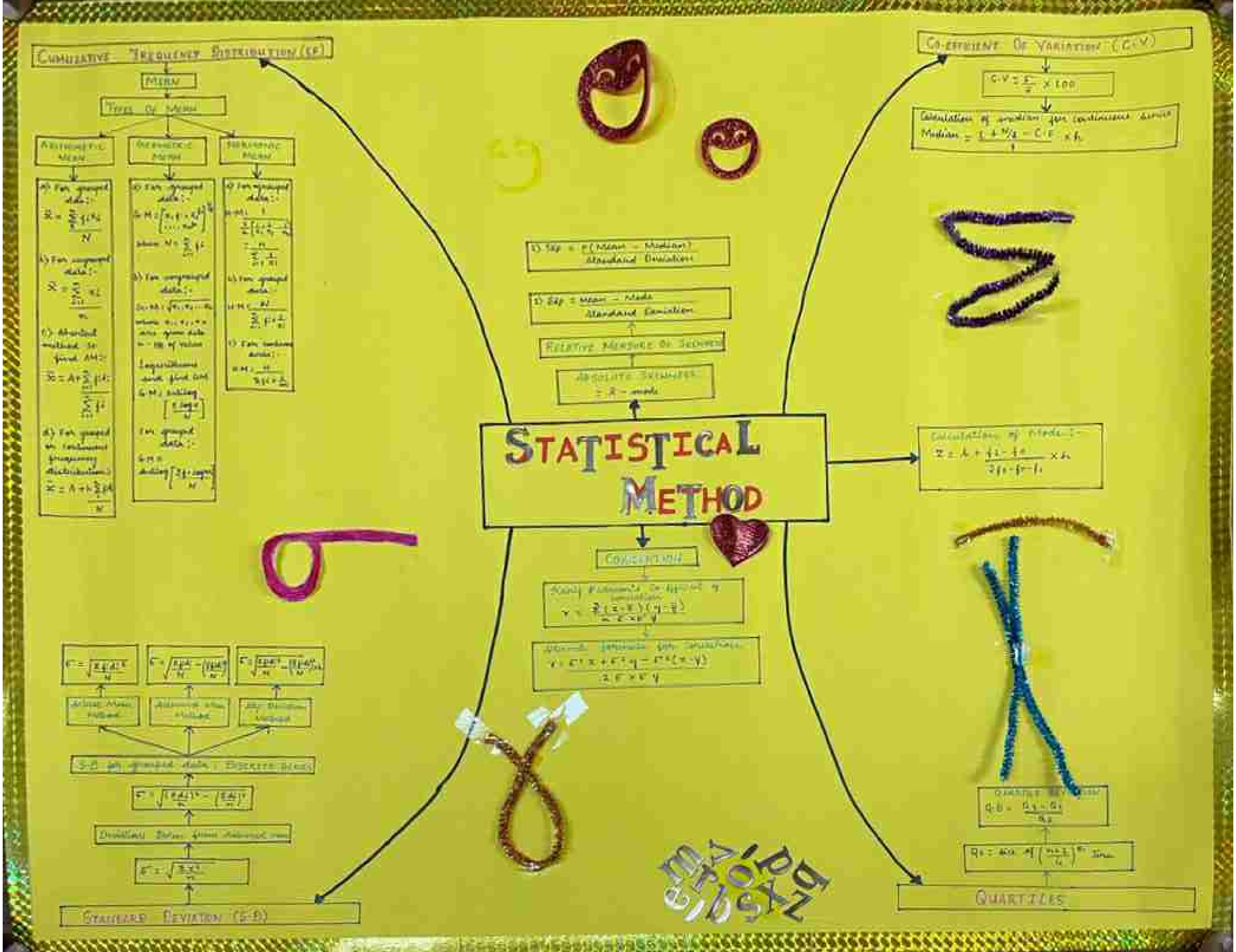
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Bisection Method

It is also called Interval Halving Method

$$x_r = \frac{a+b}{2}$$

Note :- The bisection method requires large number of iteration to achive a reasonable degree of accuracy for the root


STATISTICAL METHOD
CUMULATIVE FREQUENCY DISTRIBUTION (CF)
MEAN
TYPES OF MEAN
ARITHMETIC MEAN
GEOMETRIC MEAN
HARMONIC MEAN
STANDARD DEVIATION (S.D)
CO-EFFICIENT OF VARIATION (C.V)
Calculation of mode :-
CORRELATION
RELATIVE MEASURE OF SKEWNESS
ABSOLUTE SKEWNESS
Standard Deviation
QUARTILES

System Of Linear Equations

Introduction:

Linear Equations can be solved in different methods. When there are two different Equations we can solve by using Graphical method or matrix method or Simultaneous linear Equation methods.

When there are three different Equations we can solve it by using Gauss Elimination method, Gauss-Siedal method or Jacobi method.

Gauss Elimination Method

This method is illustrated by considering three independent Eq^{n}s in three unknowns. This method is based on Elimination of unknown by combining Eq^{n}s such that the 'n' Eq^{n}s in 'n' unknowns are reduced to an Equivalent upper triangular System which could be solved by back Substitution.

Let us consider

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \quad \ldots\ldots (1)$$

The System (1) is Equivalent to the matrix Equation $AX = B$ .... (2), where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

The method aims in reducing the Co-efficient matrix A, to upper triangular matrix

$$[A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_2 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix}$$

Step 1: We use element ($a_1 \neq 0$) to make the Elements a_2 & a_3 zero by Elementary row transformation.

$$[A:B] = \begin{vmatrix} a_1 & b_1 & c_1 & : & d_1 \\ 0 & b_2' & c_2' & : & d_2' \\ 0 & b_3' & c_3' & : & d_3' \end{vmatrix} \quad \ldots\ldots (3)$$

Step 2: We use element $b_2' \neq 0$ to make Element b_3' zero by Elementary row transform

$$[A:B] = \begin{vmatrix} a_1 & b_1 & c_1 & : & d_1 \\ 0 & b_2 & c_2 & : & d_2 \\ 0 & 0 & c_3 & : & d_3 \end{vmatrix} \quad \ldots\ldots (4)$$

from Eq^n (4) we get

$$a_1x + b_1y + c_1z = d_1$$
$$b_2'y + c_2'z = d_2'$$
$$c_3''z = d_3''$$

We get z from the last Eq^n & back substitution we get x & y of Equation.

Gauss-Jacobis method

Consider the System of Eq^{n}s

$$\begin{bmatrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{bmatrix}$$

To solve in this method we may have to rearrange the given System of Eq^{n}s to meet its requirement.

We have $x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$ ..... (1)

$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z) \ldots\ldots (2)$$

$$z = \frac{1}{c_3}(d_3 - a_3x - b_3z) \ldots\ldots (3)$$

Let $x = x_0$, $y = y_0$, $z = z_0$ using these values (first iterative) is

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0), \quad z_1 = \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0)$$

$$y_1 = \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0)$$

Similarly the Second iterative values are

$$x_2 = \frac{1}{a_1}(d_1 - b_1y_1 - c_1z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

The above process is repeated until two Consecutive iterative values are Same.

Gauss-Seidel method

It is the modification of jacobi's method. Consider the Eq^{n}s

$$\begin{bmatrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{bmatrix}$$

Let, $x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$ .... (1)

$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z) \ldots\ldots (2)$$

$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y) \ldots\ldots (3)$$

for initial values let $x = x_0$, $y = y_0$, $z = z_0$

Then $x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$

$$y_1 = \frac{1}{b_2}(d_2 - b_2x_1 - c_2z_0)$$

$$z_1 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

The Second iterative values can be calculated by Substituting iterative values of (1) (2) (3)

$$x_2 = \frac{1}{a_1}(d_1 - b_1y_1 - c_1z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2x_2 - c_2z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3x_2 - b_3y_2)$$

The Same process will go on until a iterative values of x, y & z are Same.

Numerical statatistical methods

I Arithmetic mean

$\bar{x} = \frac{\Sigma x}{N}$

where $\bar{x}$ = Arithmetic mean or average
Σx = sum of values of variable
x = value of variable
N = Total no. of observation or item

Direct method:

In case of individual series	In case of discrete	In case of continuous
$\bar{x} = \frac{\Sigma x}{N}$	$\bar{x} = \frac{\Sigma f x}{N}$ f is frequency N is sum of all the frequency $\Sigma f x$ is sum of (fx)	$\bar{x} = \frac{\Sigma f x}{N}$ f is frequency N is sum of all frequency $\Sigma f x$ is sum of (fx)

② Short cut method

In case of individual series	In case of discrete series	In case of continuous series
$\bar{x} = A + \frac{\Sigma d}{N}$ $d = x - A$ A is assumed mean Σd is sum of d	$\bar{x} = A + \frac{\Sigma f d}{N}$ f is frequency $\Sigma f d$ is sum of (fd)	$\bar{x} = A + \frac{\Sigma f d}{N}$

③ Step-deviation method

In case of individual series	In case of discrete series	In case of continuous series
$\bar{x} = A + \frac{\Sigma d}{N} \times i$ A is assumed mean $d = \frac{x - A}{i}$, Σd is sum of d i = common magnitude	$\bar{x} = A + \frac{\Sigma f d}{N} \times i$ f is frequency N is sum of frequency fd is sum of values (fd)	$\bar{x} = A + \frac{\Sigma f d}{N} \times i$

II Geometric mean (G.M)

① Individual series

$G.M = (x_1 \times x_2 \times x_3 \ldots x_n)^{1/n} = \sqrt[n]{x_1 x_2 \ldots x_n}$

② Discrete series

$G.M = \left(\frac{\Sigma f(\log x)}{N}\right)$

N is observation
f is frequency
Σf is sum of all frequency
$\Sigma f(\log x)$ = sum of all $(f \log x)$

③ continuous series

$G.M = \left(\frac{\Sigma f(\log x)}{N}\right)$

III Harmonic mean (H.M)

① Individual series:

$H.M = \frac{N}{\Sigma \frac{1}{x}}$

x is variable
$\frac{1}{x}$ is
N is observation
$\Sigma \frac{1}{x}$ is sum of all $\frac{1}{x}$

② Discrete series

$H.M = \frac{N}{\Sigma f \frac{1}{x}}$ or

③ continuous series

$H.M = \frac{N}{\Sigma f \times \frac{1}{x}}$

$\Sigma f \times \frac{1}{x}$ is the sum of all variable × (f value $\frac{1}{x}$)

→ Median

① Individual series

If the number of observations is odd then median is the middle value after the value have

in case of even number of observations, two middle terms is obtained by taking the A.M of the middle term

② Discrete series

$median(m) = \text{Size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$

N is sum of all frequency

③ continuous series

$m = \text{value of } \left(\frac{N}{2}\right)^{th} \text{ item}$

$median(m) = L + \frac{\frac{N}{2} - c}{f} \times i$

L is lower limit
i is class interval
f is frequency
c is cummulative frequency

→ Quartile:

① for discrete

$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ term}$

$Q_3 = \text{size of } 3\left(\frac{N+1}{4}\right)^{th} \text{ term}$

② for continuous

$Q_1 = \text{size of } \left(\frac{N}{4}\right)^{th} \text{ term}$

$Q_3 = \text{size of } \left(\frac{3N}{4}\right)^{th} \text{ term}$

Quartile: $\frac{Q_3 - Q_1}{2}$

→ mode

① for Individual series

The terms are arranged in any order, depending if each term of the value that occurs maximum time is known as individual term

② for discrete

here the mode is known by inspection method only here that variable is the mode where frequency is highest

③ continuous

mode
$Z = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

L is lowest limit
f_1 is frequency
f_2 is frequency of succeeding
f_0 is frequency of preceding
i is length of

→ Relation b/w mean, mode and median

$Z = 3M - 2\bar{x}$

→ STANDARD DEVIATION ~~a) direct method~~ (Individual)

a) direct method

$\sigma = \sqrt{\frac{\Sigma d^2}{N}}$, $d = (x - \bar{x})$

b) Assumed mean

$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$, $d = (x - A)$

c) Step deviation

$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2} \times i$

② Discrete series

a) direct

$\sigma = \sqrt{\frac{\Sigma f d^2}{N}}$

b) Assumed mean

$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}$

c) ~~mean~~ step-deviation

$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2} \times i$

③ continuous

a) direct

$\sigma = \sqrt{\frac{\Sigma f d^2}{N}}$

b) Assumed mean

$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}$

c) step deviation

$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2} \times i$

→ Coefficient of variation

$C.V = \frac{\sigma}{\bar{x}} \times 100$

→ Skewness (Karl Pearson)

Skewness based on mean and mode

① Absolute skewness = $\bar{x} - Z$

when skewness based on quartile

② $Q_3 + Q_1 - 2\,median$

→ Relative measures of skewness

① $S_{kp} = \frac{mean - mode}{S.D} = \frac{\bar{x} - Z}{\sigma}$

② when mode is ill defined

$S_{kp} = \frac{3(mean - median)}{S.D}$

or $S_{kp} = \frac{3(\bar{x} - M)}{\sigma}$

→ Correlation (Karl Pearson)

① Direct

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$

② Short cut method

$r = \frac{N\Sigma dxdy - \Sigma dx\Sigma dy}{\ldots}$

→ Spearman's correlation

$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$d = (x - y)$

INTERPOLATION & NUMERICAL DIFFERENTIATION

NEWTON'S FORMULAE FOR INTERPOLATION USING DIFFERENCES

FORWARD

$$P(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{h^2 2!}(x-x_0)(x-x_1) + \dots + \frac{\Delta^n y_0}{h^n n!}(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

BACKWARD

$$y_n(x) = y_0 + \frac{\nabla y_n}{h}(x-x_n) + \frac{\nabla^2 y_n}{h^2 2!}(x-x_n)(x-x_{n-1}) + \dots + \frac{\nabla^n y_n}{h^n n!}(x-x_n)(x-x_{n-1})\dots(x-x_1)$$

INTERPOLATING POLYNOMIAL

LAGRANGE FORM

$$P(x) = \sum_{i=0}^{n} l_i(x) f(x_i)$$

$i=0$, $l_0(x) = \frac{x-x_1}{x_0-x_1}$

$i=1$, $l_1(x) = \frac{x-x_0}{x_1-x_0}$

NEWTON'S FORM

$$y_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots + a_n(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

where $a_i = y_i$

INVERSE INTERPOLAT^N

LAGRANGE'S METHOD

$$x = \frac{(y-y_1)(y-y_2)\dots(y-y_n)\,x_0}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)} + \frac{(y-y_0)(y-y_2)(y-y_3)\dots(y-y_n)\,x_1}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\dots(y_1-y_n)} + \dots + \frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})\,x_n}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})}$$

NUMERICAL DIFFERENTIATION

NON-UNIFORM

LINEAR

$$P(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$$

$l_0(x) = \frac{x-x_1}{x_0-x_1}$, $l_1(x) = \frac{x-x_0}{x_1-x_0}$

QUADRATIC — 1st Order

$$f'(x_0) = \frac{2x_0-x_1-x_2}{(x_0-x_1)(x_0-x_2)} f_0 + \frac{x_0-x_2}{(x_1-x_0)(x_1-x_2)} f_1 + \frac{x_0-x_1}{(x_2-x_0)(x_2-x_1)} f_2$$

2nd Order

$$f''(x) = \frac{2f_0}{(x_0-x_1)(x_0-x_2)} + \frac{2f_1}{(x_1-x_0)(x_1-x_2)} + \frac{2f_2}{(x_2-x_0)(x_2-x_1)}$$

UNIFORM

LINEAR

$$f'(x_0) = \frac{f(x_1)-f(x_0)}{h}$$

Quadratic – 1st order

$$f'(x_0) = \frac{1}{2h}[-3f_0 + 4f_1 - f_2]$$

2nd order

$$f''(x_0) = \frac{1}{h^2}[f(x_0) - 2f(x_1) + f(x_2)]$$

NUMERICAL INTEGRATION

TRAPEZOIDAL RULE:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}[(y_0+y_n) + 2(y_1+y_2+\dots+y_{n-1})]$$

SIMPSON'S ONE-THIRD RULE:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}[(y_0+y_n) + 2(y_2+y_4+\dots+y_{n-2}) + 4(y_1+y_3+y_5+\dots+y_{n-1})]$$

SIMPSON'S THREE-EIGHTHS RULE:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\dots+y_{n-1}) + 2(y_3+y_6+\dots+y_{n-3})]$$

DIFFERENCE TABLE

FORWARD

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		Δy_0			
x_1	y_1		$\Delta^2 y_0$		
		Δy_1		$\Delta^3 y_0$	
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$
		Δy_2		$\Delta^3 y_1$	
x_3	y_3		$\Delta^2 y_2$		
		Δy_3			
x_4	y_4				

BACKWARD

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		∇y_1			
x_1	y_1		$\nabla^2 y_2$		
		∇y_2		$\nabla^3 y_3$	
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$
		∇y_3		$\nabla^3 y_4$	
x_3	y_3		$\nabla^2 y_4$		
		∇y_4			
x_4	y_4				

DIVIDED DIFFERENCE TABLE

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	$f(x_0)$			
		$\frac{f(x_1)-f(x_0)}{x_1-x_0}$		
x_1	$f(x_1)$		$\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$	
		$\frac{f(x_2)-f(x_1)}{x_2-x_1}$		$\frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$
x_2	$f(x_2)$		$\frac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1}$	
		$\frac{f(x_3)-f(x_2)}{x_3-x_2}$		
x_3	$f(x_3)$			

FLOATING POINT REPRESENTION

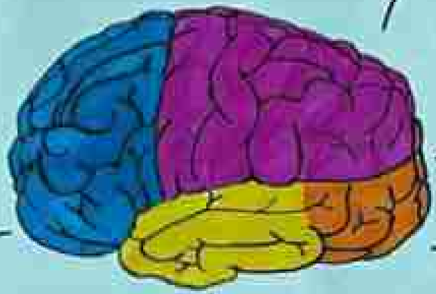
A Machine number in single precision floating point ...

For Single Precision ...

where, 0 → Positive

1 → Negative

Representation



DOUBLE PRECISION

A Machine number for Double precision floating point form is given by,

$(-1)^s \times 2^{c-1023} \times (1+f)$.

For Double Precision → $c \leq 1023$

where, 0 → Positive

1 → Negative

Representation $\boxed{s}\boxed{c}\boxed{f}$

BISECTION METHOD

The Bisection Method ... large number of iteration ... accuracy for the root. It is also called ... Method

Formula: $x_r = \frac{a+b}{2}$

Newton-Raphson Method

A better approximation to the root can be found by using an iterative process ... Newton-Raphson Method

Formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Venkatesh.B

Hitesh

Yashwanth

Team members:-
Rushi-H
Ingham-J
Sagar reddy.-YB

Numerical Integration:
⇓
Trapezoidal:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\dots+y_{n-1})\right]$$

Simpson's $(1/3)^{rd}$ rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+\dots+y_{n-2}) + 4(y_1+y_3+y_5+\dots+y_{n-1})\right]$$

Simpson's $(3/8)$ rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\dots+y_{n-1}) + 2(y_3+y_6+\dots+y_{n-3})\right]$$

Lagrange form:

$$P(x) = \sum_{i=0}^{n} L_i(x)\, f(x_i)$$

$i=0,\ L_0(x) = \frac{x-x_1}{x_0-x_1}$

$i=1,\ L_1(x) = \frac{x-x_0}{x_1-x_0}$

Inverse Interpolation
↓
Lagrange method:

$$x = \frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\dots(y_1-y_n)}x_1 + \dots + \frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})}x_n$$

"Interpolation & numerical differentiation"

Newtons general difference divided formula:-

$$f(x) = f(x_0) + (x-x_0)\,f(x_0,x_1) + (x-x_0)(x-x_1)\,f(x_0,x_1,x_2) + \dots + (x-x_0)(x-x_1)\dots f(x_0,x_1,\dots,x_n)$$

The divided difference table:

Argument x	Entry $f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	$f(x_0)$			
		$f(x_0,x_1)$		
x_1	$f(x_1)$		$f(x_0,x_1,x_2)$	
		$f(x_1,x_2)$		$f(x_0,x_1,x_2,x_3)$
x_2	$f(x_2)$		$f(x_1,x_2,x_3)$	
		$f(x_2,x_3)$		
x_3	$f(x_3)$			

Newton forward difference table:-

[illegible]					

Newton's form:-

$$y_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots + a_n(x-x_0)\dots(x-x_{n-1})$$

where $a_0 = y_0$

Newtons backward diff table:-

[illegible]					

Numerical differentiation:
⇓
Quadratic Interpolation → 1^{st} order:

$$f'(x_0) = \frac{2x_0-x_1-x_2}{(x_0-x_1)(x_0-x_2)}f_0 + \frac{x_0-x_2}{(x_1-x_0)(x_1-x_2)}f_1 + \frac{x_0-x_1}{(x_2-x_0)(x_2-x_1)}f_2$$

2^{nd} order:

$$f''(x) = \frac{2f_0}{(x_0-x_1)(x_0-x_2)} + \frac{2f_1}{(x_1-x_0)(x_1-x_2)} + \frac{2f_2}{(x_2-x_0)(x_2-x_1)}$$